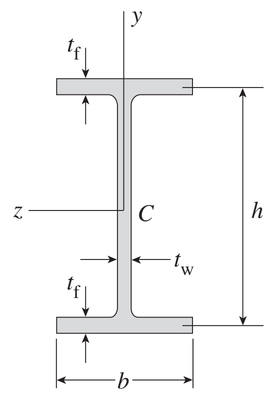
Problem 6.8-3 A beam of wide-flange shape has the cross section shown in the figure. The dimensions are $b = 5.25$ in., $h = 7.9$ in., $t_w = 0.25$ in., and $t_f = 0.4$ in. The loads on the beam produce a shear force $V = 6.0$ k at the cross section under consideration.

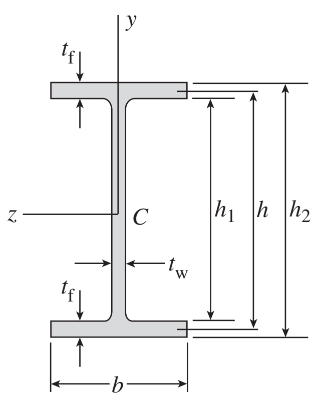
(a) Using centerline dimensions, calculate the maximum shear stress in the web of the beam.

(b) Using the more exact analysis of Section 5.10 in Chapter 5, calculate the maximum shear stress in the web of the beam and compare it with the stress obtained in part (a).



Probs. 6.8-3 and 6.8-4

Solution 6.8-3 Wide-flange beam



$b = 5.25$ in. $h = 7.9$ in. $t_w = 0.25$ in.
$t_f = 0.4$ in. $V = 6.0$ k

(a) CALCULATIONS BASED ON CENTERLINE DIMENSIONS (SECTION 6.8)

Moment of inertia (Eq. 6-59): $I_z = \dfrac{t_w h^3}{12} + \dfrac{bt_f h^2}{2}$

$I_z = 10.272 + 65.531 = 75.803 \text{ in.}^4$

Maximum shear stress in the web (Eq. 6-54):

$$\tau_{max} = \left(\frac{bt_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z} = (10.375 \text{ in.})\ (312.65 \text{ lb/in.}^3)$$

$$= 3244 \text{ psi} \quad \longleftarrow$$

(b) CALCULATIONS BASED ON MORE EXACT ANALYSIS (SECTION 5.10)

See Figure 5-38. Replace h by h_2 and t by t_w.

$h_2 = h + t_f = 8.3$ in. $h_1 = h - t_f = 7.5$ in.

Moment of inertia (Eq. 5-47):

$$I = \frac{1}{12}(bh_2^3 - bh_1^3 + t_w h_1^3)$$

$$I = \frac{1}{12}(892.51 \text{ in.}^4) = 74.376 \text{ in.}^4$$

Maximum shear stress in the web (Eq. 5-48a):

$$\tau_{max} = \frac{V}{8It_w}(bh_2^2 - bh_1^2 + t_w h_1^2)$$

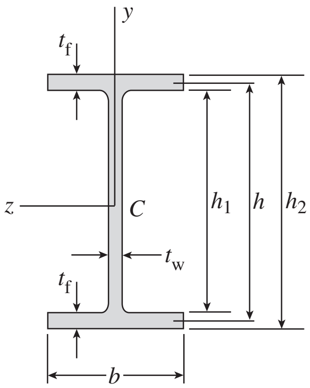
$$= (40.336 \text{ lb/in.}^5)(80.422 \text{ in.}^3)$$

$$= 3244 \text{ psi} \quad \longleftarrow$$

NOTE: Within the accuracy of the calculations, the maximum shear stresses are the same.

Problem 6.8-4 Solve the preceding problem for the following data: $b = 145$ mm, $h = 250$ mm, $t_w = 8.0$ mm, $t_f = 14.0$ mm, and $V = 30$ kN.

Solution 6.8-4 Wide-flange beam



$b = 145$ mm
$h = 250$ mm
$t_w = 8.0$ mm
$t_f = 14.0$ mm
$V = 30$ kN

(a) CALCULATIONS BASED ON CENTERLINE DIMENSIONS (SECTION 6.8)

Moment of inertia (Eq. 6-57): $I_z = \dfrac{t_w h^3}{12} + \dfrac{bt_f h^2}{2}$

$I_z = 10.417 \times 10^6 \text{ mm}^4 + 63.438 \times 10^6 \text{ mm}^4$
$= 73.855 \times 10^6 \text{ mm}^4$

Maximum shear stress in the web (Eq. 6-54):

$$\tau_{max} = \left(\frac{bt_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z}$$
$$= (316.25 \text{ mm})(0.050775 \text{ N/mm}^3)$$
$$= 16.06 \text{ MPa} \longleftarrow$$

(b) CALCULATIONS BASED ON MORE EXACT ANALYSIS (SECTION 5.10)

See Figure 5-38. Replace h by h_2 and t by t_w.
$h_2 = h + t_f = 264$ mm $\quad h_1 = h - t_f = 236$ mm

Moment of inertia (Eq. 5-47):

$$I = \frac{1}{12}(bh_2^3 - bh_1^3 + t_w h_1^3)$$

$$I = \frac{1}{12}(867.20 \times 10^6 \text{ mm}^4) = 72.267 \times 10^6 \text{ mm}^4$$

Maximum shear stress in the web (Eq. 5-48a):

$$\tau_{max} = \frac{V}{8It_w}(bh_2^2 - bh_1^2 + t_w h_1^2)$$
$$= (6.4864 \times 10^{-6} \text{ N/mm}^5)(2.4756 \times 10^6 \text{ mm}^2)$$
$$= 16.06 \text{ MPa} \longleftarrow$$

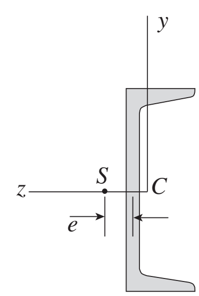
NOTE: Within the accuracy of the calculations, the maximum shear stresses are the same.

Shear Centers of Thin-Walled Open Sections

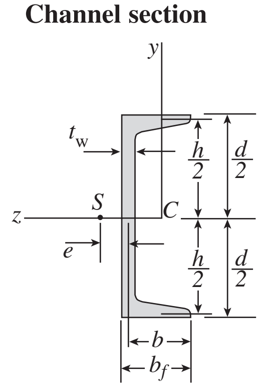
When locating the shear centers in the problems for Section 6.9, assume that the cross sections are thin-walled and use centerline dimensions for all calculations and derivations.

Problem 6.9-1 Calculate the distance e from the centerline of the web of a C 12 × 20.7 channel section to the shear center S (see figure).

(*Note:* For purposes of analysis, consider the flanges to be rectangles with thickness t_f equal to the average flange thickness given in Table E-3, Appendix E.)



Solution 6.9-1 Channel section



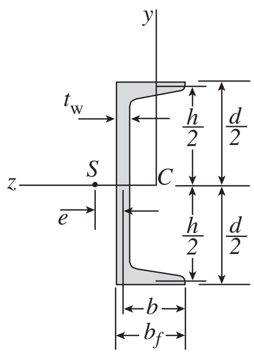
C 12 × 20.7 $\quad d = 12.00$ in. $\quad t_w = 0.282$ in.
$b_f = 2.942$ in. $\quad t_f$ = average flange thickness
$t_f = 0.501$ in. $\quad b = b_f - tw/2 = 2.801$ in.
$h = d - t_f = 11.499$ in.

Eq. (6-65): $c = \dfrac{3b^2 t_f}{ht_w + 6bt_f} = 1.011$ in. $\longleftarrow$

Problem 6.9-2 Calculate the distance e from the centerline of the web of a C 8 × 18.75 channel section to the shear center S (see figure).

(*Note:* For purposes of analysis, consider the flanges to be rectangles with thickness t_f equal to the average flange thickness given in Table E-3, Appendix E.)

Solution 6.9-2 Channel section



C 8 × 18.75 $d = 8.00$ in. $t_w = 0.487$ in.

$b_f = 2.527$ in. t_f = average flange thickness

$t_f = 0.390$ in. $b = b_f - tw/2 = 2.284$ in.

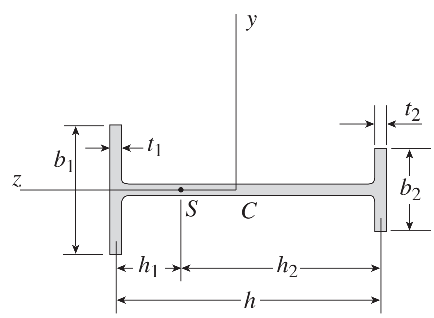
$h = d - t_f = 7.610$ in.

Eq. (6-65): $c = \dfrac{3b^2t_f}{ht_w + 6bt_f} = 0.674$ in. ←

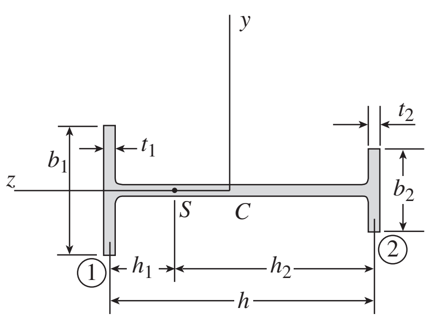
Problem 6.9-3 The cross section of an unbalanced wide-flange beam is shown in the figure. Derive the following formula for the distance h_1 from the centerline of one flange to the shear center S:

$$h_1 = \frac{t_2b_2^3h}{t_1b_1^3 + t_2b_2^3}$$

Also, check the formula for the special cases of a T-beam ($b_2 = t_2 = 0$) and a balanced wide-flange beam ($t_2 = t_1$ and $b_2 = b_1$).



Solution 6.9-3 Unbalanced wide-flange beam



FLANGE 1:

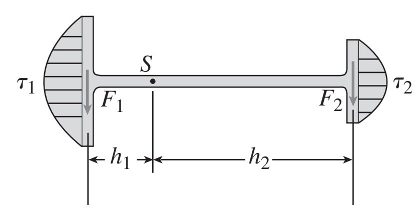
$$\tau_1 = \frac{VQ}{I_zt_1}$$

$$Q = (b_1/2)(t_1)(b_1/4) = \frac{t_1b_1^2}{8}$$

$$\tau_1 = \frac{Vb_1^2}{8I_z}$$

$$F_1 = \frac{2}{3}(\tau_1)(b_1)(t_1) = \frac{Vt_1b_1^3}{12I_z}$$

FLANGE 2:



$$F_2 = \frac{Vt_2b_2^3}{12I_z}$$

Shear force V acts through the shear center S.

$\therefore \sum M_s = F_1h_1 - F_2h_2 = 0$

or $(t_1b_1^3)h_1 = (t_2b_2^3)h_2$ (1)

$h_1 + h_2 = h$ (2)

Solve Eqs. (1) and (2): $h_1 = \dfrac{t_2b_2^3h}{t_1b_1^3 + t_2b_2^3}$ ←

T-BEAM

$b_2 = t_2 = 0$;

$\therefore h_1 = 0$ ←

WIDE-FLANGE BEAM

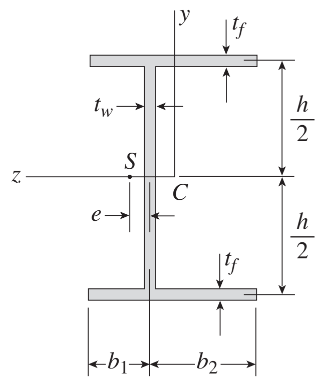
$t_2 = t_1$ and $b_2 = b_1$;

$\therefore h_1 = h/2$ ←

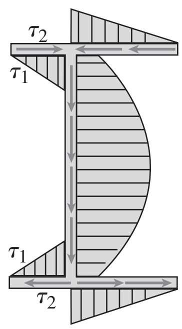
Problem 6.9-4 The cross section of an unbalanced wide-flange beam is shown in the figure. Derive the following formula for the distance e from the centerline of the web to the shear center S:

$$e = \frac{3t_f(b_2^2 - b_1^2)}{ht_w + 6t_f(b_1 + b_2)}$$

Also, check the formula for the special cases of a channel section ($b_1 = 0$ and $b_2 = b$) and a doubly symmetric beam ($b_1 = b_2 = b/2$).



Solution 6.9-4 Unbalanced wide-flange beam



$$\tau_1 = \frac{VQ}{It_f} = \frac{b_1 hV}{2I_z} \qquad \tau_2 = \frac{b_2 hV}{2I_z}$$

$$F_1 = \frac{b_1\tau_1 t_f}{2} = \frac{b_1^2 ht_f V}{4I_z} \qquad F_2 = \frac{b_2^2 ht_f V}{4I_z} \qquad F_3 = V$$

Shear force V acts through the shear center S.

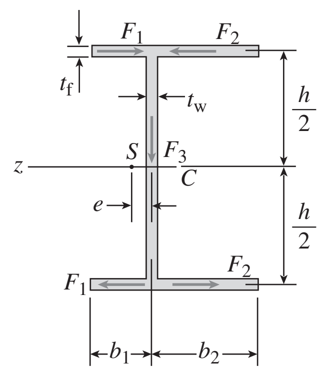
$$\therefore \sum M_s = -F_3 e - F_1 h + F_2 h = 0$$

$$e = \frac{F_2 h - F_1 h}{F_3} = \frac{h^2 t_f}{4I_z}(b_2^2 - b_1^2)$$

$$I_z = \frac{t_w h^3}{12} + 2(b_1 + b_2)(t_f)\left(\frac{h}{2}\right)^2$$

$$= \frac{h^2}{12}[ht_w + 6t_f(b_1 + b_2)]$$

$$e = \frac{3t_f(6_2^2 - 6_1^2)}{ht_w + 6t_f(b_1 + b_2)} \quad \longleftarrow$$



CHANNEL SECTION ($b_1 = 0$, $b_2 = b$)

$$e = \frac{3b^2 t_f}{ht_w + 6bt_f} \qquad \text{(Eq. 6-65)}$$

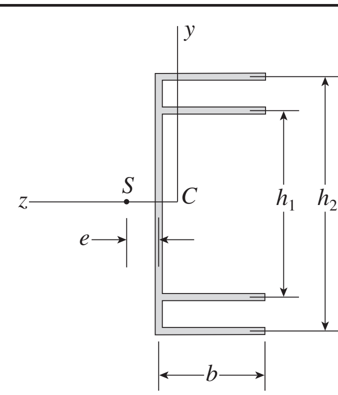
DOUBLY SYMMETRIC BEAM ($b_1 = b_2 = b/2$)

$e = 0$ (Shear center coincides with the centroid)

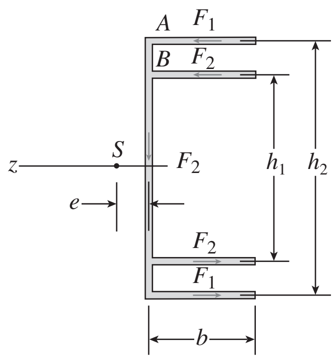
Problem 6.9-5 The cross section of a channel beam with double flanges and constant thickness throughout the section is shown in the figure.

Derive the following formula for the distance e from the centerline of the web to the shear center S:

$$e = \frac{3b^2(h_1^2 + h_2^2)}{h_2^3 + 6b(h_1^2 + h_2^2)}$$



Solution 6.9-5 **Channel beam with double flanges**



t = thickness

$$\tau_A = \frac{VQ_A}{I_z t} = \frac{V(bt)\left(\frac{h_2}{2}\right)}{I_z t} = \frac{bh_2 V}{2I_z}$$

$$F_1 = \frac{1}{2}\tau_A bt = \frac{b^2 h_2 tV}{4I_z}$$

$$\tau_B = \frac{bh_1 V}{2I_z} \qquad F_2 = \frac{b^2 h_1 tV}{4I_z}$$

$$F_3 = V$$

Shear force V acts through the shear center S.

$$\therefore \sum M_s = -F_3 e + F_1 h_2 + F_2 h_1 = 0$$

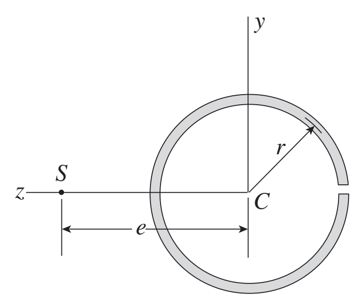
$$e = \frac{F_2 h_1 + F_1 h_2}{F_3} = \frac{b^2 t}{4I_z}(h_1^2 + h_2^2)$$

$$I_z = \frac{th_2^3}{12} + 2\,[bt(h_2/2)^2 + bt(h_1/2)^2]$$

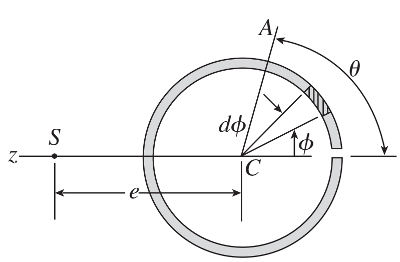
$$= \frac{t}{12}\,[h_2^3 + 6b(h_1^2 + h_2^2)]$$

$$e = \frac{3b^2(h_1^2 + h_2^2)}{h_2^3 + 6b(h_1^2 + h_2^2)} \quad \longleftarrow$$

Problem 6.9-6 The cross section of a slit circular tube of constant thickness is shown in the figure. Show that the distance e from the center of the circle to the shear center S is equal to $2r$.



Solution 6.9-6 **Slit circular tube**



r = radius
t = thickness

$$\tau_A = \frac{VQ_A}{I_z t} = \frac{Vr^2(1 - \cos\theta)}{I_z}$$

$$I_z = \pi r^3 t$$

$$Q_A = \int y\,dA$$

$$= \int_o^\theta (r\sin\phi)rt\,d\phi$$

$$= r^2 t(1 - \cos\theta)$$

$$\tau_A = \frac{V(1 - \cos\theta)}{\pi rt}$$

At point A: $dA = rt\,d\theta$

T_C = moment of shear stresses about center C.

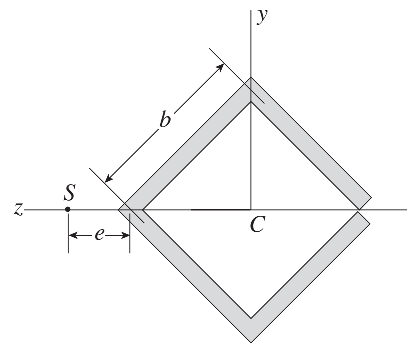
$$T_C = \int \tau_A r\,dA = \int_o^{2\pi} \frac{Vr}{\pi}(1 - \cos\theta)\,d\theta = 2Vr$$

Shear force V acts through the shear center S. Moment of the shear force V about any point must be equal to the moment of the shear stresses about that same point.

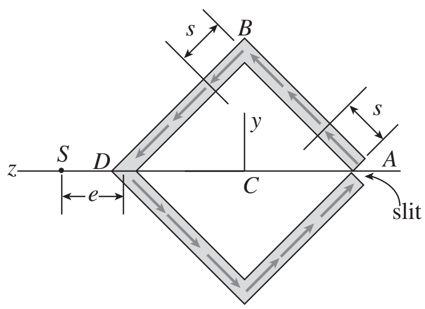
$$\therefore \sum M_C = Ve = T_C \qquad e = \frac{T_C}{V} = 2r \quad \longleftarrow$$

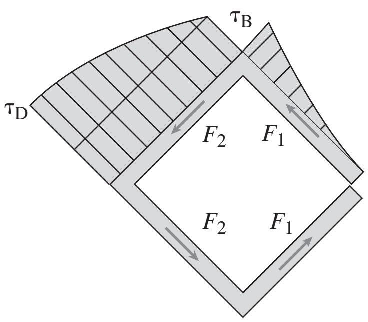
Problem 6.9-7 The cross section of a slit square tube of constant thickness is shown in the figure. Derive the following formula for the distance e from the corner of the cross section to the shear center S:

$$e = \frac{b}{2\sqrt{2}}$$



Solution 6.9-7 Slit square tube





b = length of each side

t = thickness

$$\tau = \frac{VQ}{I_z t}$$

FROM A TO B:

$$Q = \frac{tS^2}{2\sqrt{2}}$$

At A: $Q = 0 \quad \tau_A = 0$

At B: $Q = \dfrac{tb^2}{2\sqrt{2}}$

$$\tau_B = \frac{b^2 V}{2\sqrt{2} I_z}$$

$$F_1 = \frac{\tau_B bt}{3} = \frac{b^3 tV}{6\sqrt{2} I_z}$$

FROM B TO D:

$$Q = bt\left(\frac{b}{2\sqrt{2}}\right) + St\left(\frac{b}{\sqrt{2}} - \frac{S}{2\sqrt{2}}\right)$$

$$= \frac{tb^2}{2\sqrt{2}} + \frac{tS}{2\sqrt{2}}(2b - S)$$

$$\tau = \frac{VQ}{I_z t} = \frac{V}{I_z}\left[\frac{b^2}{2\sqrt{2}} + \frac{S}{2\sqrt{2}}(2b - S)\right]$$

At B: $\tau_B = \dfrac{b^2 V}{2\sqrt{2} I_z}$ At D: $\tau_B = \dfrac{b^2 V}{\sqrt{2} I_z}$

$$F_2 = \tau_B bt + \frac{2}{3}(\tau_D - \tau_B)\, bt = \frac{5tb^3 V}{6\sqrt{2} I_z}$$

Shear force V acts through the shear center S.

$\therefore \sum M_s = 0$

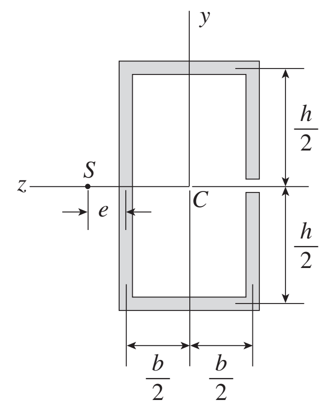
$$2(F_1/\sqrt{2})(b\sqrt{2} + e) + 2(F_2/\sqrt{2})(e) = 0$$

Substitute for F_1 and F_2 and solve for e:

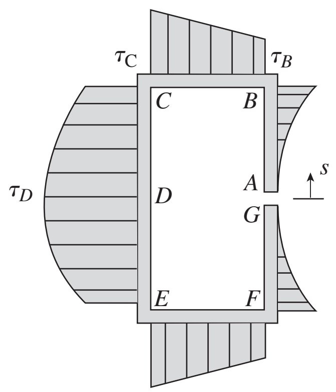
$$e = \frac{b}{2\sqrt{2}} \quad \longleftarrow$$

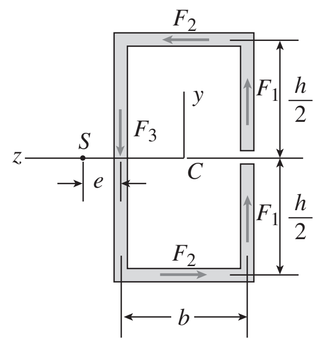
Problem 6.9-8 The cross section of a slit rectangular tube of constant thickness is shown in the figure. Derive the following formula for the distance e from the centerline of the wall of the tube to the shear center S:

$$e = \frac{b(2h + 3b)}{2(h + 3b)}$$



Solution 6.9-8 Slit rectangular tube





t = thickness

FROM A TO B: $Q = \frac{tS^2}{2} \qquad \tau = \frac{VQ}{I_z t} = \frac{S^2 V}{2 I_z}$

$$\tau_A = 0 \qquad \tau_B = \frac{h^2 V}{8 I_z} \qquad F_1 = \frac{\tau_B t}{3}\left(\frac{h}{2}\right) = \frac{th^3 V}{48 I_z}$$

FROM B TO C: $\tau_B = \frac{h^2 V}{8 I_z}$

$$Q_C = \frac{th}{2}\left(\frac{h}{4}\right) + bt\left(\frac{h}{2}\right) = \frac{th}{8}(h + 4b)$$

$$\tau_C = \frac{h(h + 4b)V}{8 I_z}$$

$$F_2 = \frac{1}{2}(\tau_B + \tau_C)\, bt = \frac{bht(h + 2b)V}{8 I_z}$$

$$\sum F_{\text{VERT}} = V \qquad F_3 - 2F_1 = V$$

$$F_3 = V\left(1 + \frac{th^3}{24 I_z}\right)$$

Shear force V acts through the shear center S.

$$\therefore \sum M_s = 0 \qquad -F_3 e + F_2 h + 2F_1(b + e) = 0$$

Substitute for F_3, F_2 and F_1 and solve for e:

$$e = \frac{bh^2 t(2h + 3b)}{12 I_z}$$

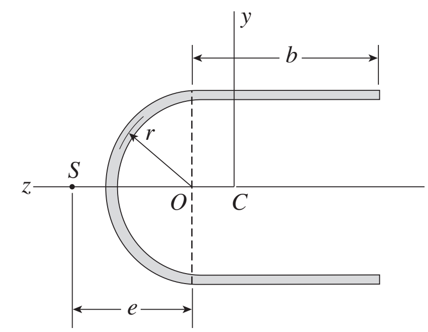
$$I_z = 2\left[\frac{1}{12} th^3 + bt\left(\frac{h}{2}\right)^2\right] = \frac{th^2}{6}(h + 3b)$$

$$e = \frac{b(2h + 3b)}{2(h + 3b)} \quad \longleftarrow$$

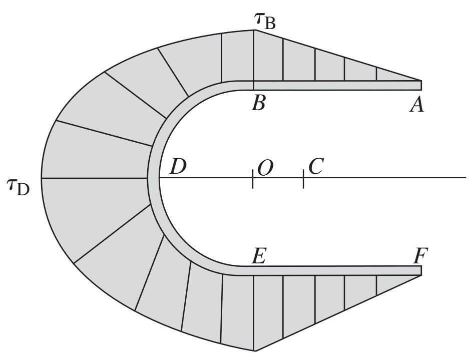
Problem 6.9-9 A U-shaped cross section of constant thickness is shown in the figure. Derive the following formula for the distance e from the center of the semicircle to the shear center S:

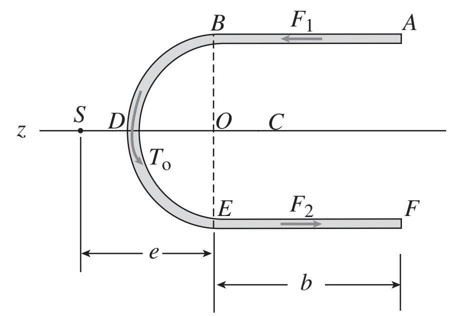
$$e = \frac{2(2r^2 + b^2 + \pi br)}{4b + \pi r}$$

Also, plot a graph showing how the distance e (expressed as the nondimensional ratio e/r) varies as a function of the ratio b/r. (Let b/r range from 0 to 2.)



Solution 6.9-9 U-shaped cross section





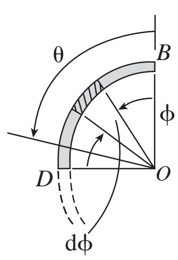
r = radius F_1 = force in AE
t = thickness F_2 = force in EF
T_0 = moment in BDE

FROM A TO B: $\tau_A = 0 \quad \tau_B = \dfrac{VQ}{I_z t} = \dfrac{V(btr)}{I_z t} = \dfrac{Vbr}{I_z}$

$$F_1 = \frac{bt\tau_B}{2} = \frac{Vb^2rt}{2I_z}$$

FROM B TO E: $Q_1 = \displaystyle\int ydA = \int_0^\theta (r\cos\phi)\, rtd\phi$

$$= r^2 t \sin\theta$$



$Q_B = btr$

$Q_e = Q_B + Q_1 = btr + r^2 t \sin\theta$

$$\tau_e = \frac{VQ_B}{I_z t} = \frac{Vr(b + r\sin\theta)}{I_z}$$

At angle θ: $dA = rtd\theta$

$$T_0 = \int \tau r dA = \int_b^\pi \tau r^2 t d\theta$$

$$= \int_0^\pi \frac{Vr^3 t(b + r\sin\theta)d\theta}{I_z}$$

$$= \frac{Vr^3 t}{I_z}(\pi b + 2r)$$

Shear force V acts through the shear center S. Moment of the shear force V about any point must be equal to the moment of the shear stresses about that same point.

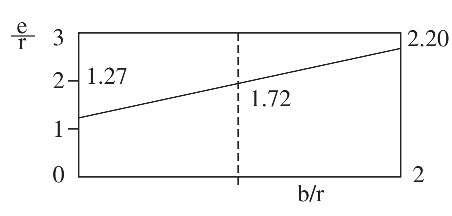
$$\therefore \sum M_0 = V_e = T_0 + F_1(2r)$$

$$e = \frac{T_0 + 2F_1 r}{V} = \frac{r^2 t}{I_z}(\pi br + 2r^2 + b^2)$$

$$I_z = \frac{\pi r^3 t}{2} + 2(btr^2) \quad e = \frac{2(2r^2 + b^2 + \pi br)}{4b + \pi r} \quad \longleftarrow$$

GRAPH

$$\frac{e}{r} = \frac{2(2 + b^2/r^2 + \pi b/r)}{4b/r + \pi}$$



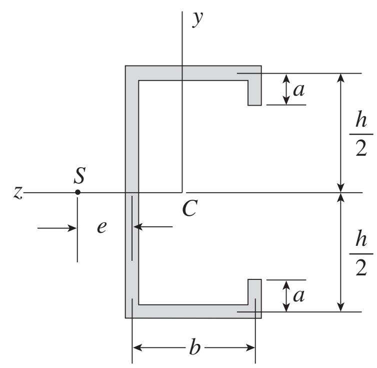
NOTE: When $b/r = 0$,

$$e/r = \frac{4}{\pi} \quad \text{(Eq. 6-73)}$$

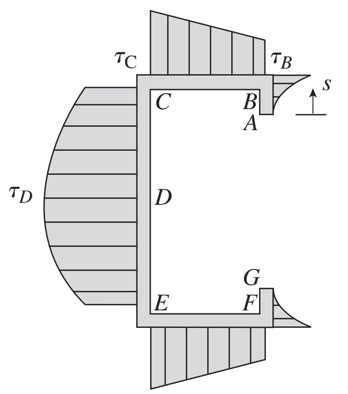
Problem 6.9-10 Derive the following formula for the distance e from the centerline of the wall to the shear center S for the C-section of constant thickness shown in the figure:

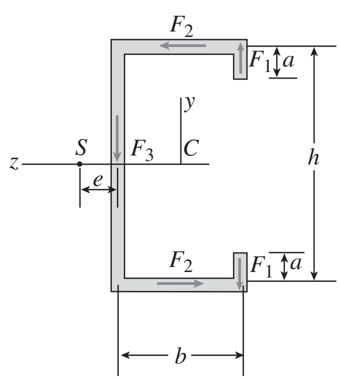
$$e = \frac{3bh^2(b + 2a) - 8ba^3}{h^2(h + 6b + 6a) + 4a^2(2a - 3h)}$$

Also, check the formula for the special cases of a channel section ($a = 0$) and a slit rectangular tube ($a = h/2$).



Solution 6.9-10 **C-section of constant thickness**





t = thickness

From A to B:

$$Q = St\left(\frac{h}{2} - a + \frac{S}{2}\right) \quad \tau = \frac{VQ}{I_z t} = S\left(\frac{h}{2} - a + \frac{S}{2}\right)\frac{V}{I_z}$$

$$\tau_A = 0 \quad \tau_B = \frac{a}{2}(h - a)\frac{V}{I_z}$$

$$F_1 = \int_0^a \tau t dS = \frac{tV}{I_z}\int_0^a S\left(\frac{h}{2} - a + \frac{S}{2}\right)dS$$

$$= \frac{a^2 t(3h - 4a)V}{12 I_z}$$

From B to C:

$$\tau_B = \frac{a}{2}(h - a)\frac{V}{I_z} \quad Q_C = at\left(\frac{h}{2} - \frac{a}{2}\right) + bt\left(\frac{h}{2}\right)$$

$$= \frac{at}{2}(h - a) + \frac{bht}{2}$$

$$\tau_C = \left[\frac{a}{2}(h - a) + \frac{bh}{2}\right]\frac{V}{I_z}$$

$$F_2 = \frac{1}{2}(\tau_B + \tau_C)\,bt = \frac{bt}{4}[2a(h - a) + bh]\frac{V}{I_z}$$

From C to E:

$$\sum F_{\text{VERT}} = V \quad F_3 - 2F_1 = V$$

$$F_3 = V\left[1 + \frac{a^2 t(3h - 4a)}{6 I_z}\right]$$

Shear force V acts through the shear center S.

$$\therefore \sum M_S = 0 \quad -F_3(e) + F_2 h + 2F_1(b + e) = 0$$

Substitute for F_1, F_2, and F_3 and solve for e:

$$e = \frac{bt[3h^2(b + 2a) - 8a^3]}{12 I_z}$$

$$I_z = 2\left(\frac{1}{12}th^3\right) + 2bt\left(\frac{h}{2}\right)^2 - \frac{6}{12}(h - 2a)^3$$

$$= \frac{t}{12}[h^2(h + 6b + 6a) + 4a^2(2a - 3h)]$$

$$e = \frac{3bh^2(b + 2a) - 8ba^3}{h^2(h + 6b + 6a) + 4a^2(2a - 3h)} \quad \longleftarrow$$

Channel section ($a = 0$)

$$e = \frac{3b^2}{h + 6b}$$ (agrees with Eq. 6-65 when $t_f = t_w$)

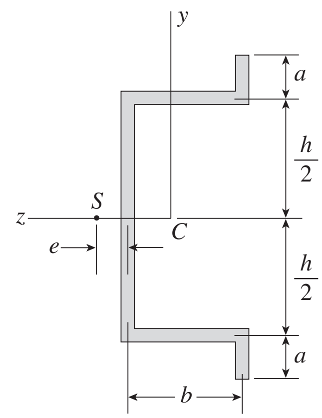
Slit rectangular tube ($a = h/2$)

$$e = \frac{b(2h + 3b)}{2(h + 3b)}$$ (agrees with the result of Prob. 6.9-8)

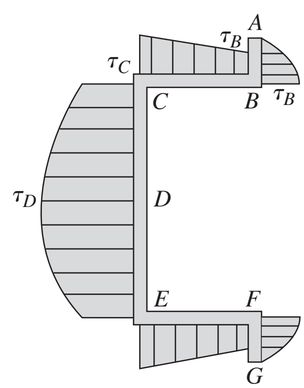
Problem 6.9-11 Derive the following formula for the distance e from the centerline of the wall to the shear center S for the hat section of constant thickness shown in the figure:

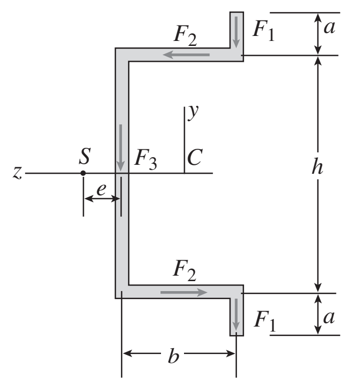
$$e = \frac{3bh^2(b + 2a) - 8ba^3}{h^2(h + 6b + 6a) + 4a^2(2a + 3h)}$$

Also, check the formula for the special case of a channel section ($a = 0$).



Solution 6.9-11 Hat section of constant thickness





t = thickness

FROM A TO B: $Q = St\left(\frac{h}{2} + a - \frac{S}{2}\right)$

$$\tau = \frac{VQ}{I_z t} = S\left(\frac{h}{2} + a - \frac{S}{2}\right)\frac{V}{I_z}$$

$$\tau_A = 0 \quad \tau_B = \frac{a}{2}(h + a)\frac{V}{I_z}$$

$$F_1 = \int_0^a \tau t dS = \frac{tV}{I_z}\int_0^a S\left(\frac{h}{2} + a - \frac{S}{2}\right)dS$$

$$= \frac{a^2 t(3h + 4a)V}{12 I_z}$$

FROM B TO C: $\tau_B = \frac{a}{2}(h + a)\frac{V}{I_z}$

$$Q_C = at\left(\frac{h}{2} + \frac{a}{2}\right) + bt\left(\frac{h}{2}\right) = \frac{at}{2}(h + a) + \frac{bht}{2}$$

$$\tau_C = \left[\frac{a}{2}(h + a) + \frac{bh}{2}\right]\frac{V}{I_z}$$

$$F_2 = \frac{1}{2}(\tau_B + \tau_C)\,bt = \frac{bt}{4}\left[2a(h + a) + bh\right]\frac{V}{I_z}$$

FROM C TO E:

$$\sum F_{\text{VERT}} = V \quad F_3 + 2F_1 = V$$

$$F_3 = V\left[1 - \frac{a^2 t(3h + 4a)}{6 I_z}\right]$$

Shear force V acts through the shear center S.

$$\therefore \sum M_S = 0 \quad -F_3 e + F_2 h - 2F_1(b + e) = 0$$

Substitute for F_1, F_2, and F_3 and solve for e:

$$e = \frac{bt[3h^2(b + 2a) - 8a^3]}{12 I_z}$$

$$I_z = \frac{1}{12}th^3 + 2bt\left(\frac{h}{2}\right)^2 + \frac{t}{12}(h + 2a)^3 - \frac{1}{12}th^3$$

$$= \frac{t}{12}\left[h^2(h + 6b + 6a) + 4a^2(2a + 3h)\right]$$

$$e = \frac{3bh^2(b + 2a) - 8ba^3}{h^2(h + 6b + 6a) + 4a^2(2a + 3h)} \quad \longleftarrow$$

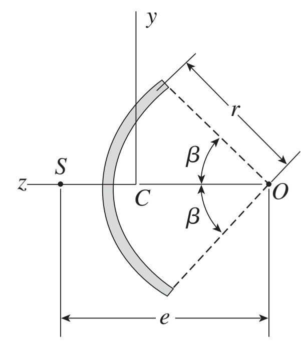
CHANNEL SECTION ($a = 0$)

$$e = \frac{3b^2}{h + 6b} \quad \text{(agrees with Eq. 6-65 when } t_f = t_w\text{)}$$

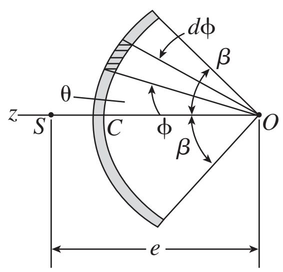
Problem 6.9-12 A cross section in the shape of a circular arc of constant thickness is shown in the figure. Derive the following formula for the distance e from the center of the arc to the shear center S:

$$e = \frac{2r(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta\cos\beta}$$

in which β is in radians. Also, plot a graph showing how the distance e varies as β varies from 0 to π.



Solution 6.9-12 **Circular arc**



t = thickness
r = radius
At angle θ:

$$Q = \int y\,dA = \int_{\theta}^{\beta} (r\sin\phi) rt\,d\phi = r^2 t(\cos\theta - \cos\beta)$$

$$\tau = \frac{VQ}{I_z t} = \frac{Vr^2(\cos\theta - \cos\beta)}{I_z}$$

$$I_z = \int y^2 dA = \int_{-\beta}^{\beta} (r\sin\phi)^2 rt\,d\phi = r^3 t(\beta - \sin\beta\cos\beta)$$

$$\tau = \frac{V(\cos\theta - \cos\beta)}{rt(\beta - \sin\beta\cos\beta)}$$

T_0 = moment of shear stresses

At angle θ, $dA = rtd\theta$

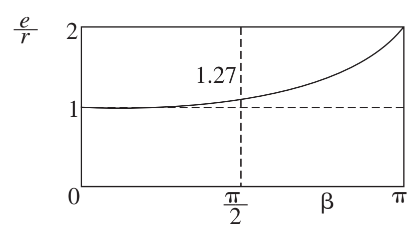
$$T_0 = \int \tau r\,dA = \int_{-\beta}^{\beta} \frac{V(\cos\theta - \cos\beta)}{t(\beta - \sin\beta\cos\beta)} rt\,d\theta = \frac{2Vr(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta\cos\beta}$$

Shear force V acts through the shear center S. Moment of the shear force V about any point must be equal to the moment of the shear stresses about that same point.

$$\therefore \sum M_0 = Ve = T_0 \quad e = T_0/V$$

$$e = \frac{2r(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta\cos\beta} \quad \longleftarrow$$

Graph



$$\frac{e}{r} = \frac{2(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta\cos\beta}$$

Semicircular arc ($\beta = \pi/2$):

$$\frac{e}{r} = \frac{4}{\pi} \quad \text{(Eq. 6-73)}$$

Slit circular arc ($\beta = \pi$):

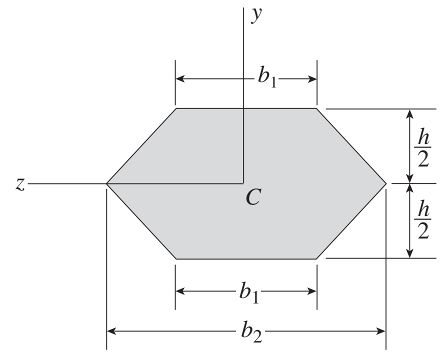
$$\frac{e}{r} = 2 \text{ (Prob. 6.9-6)}$$

Elastoplastic Bending

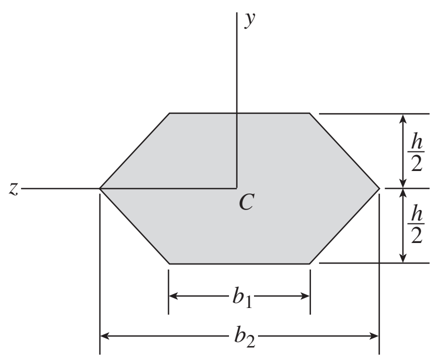
The problems for Section 6.10 are to be solved using the assumption that the material is elastoplastic with yield stress σ_Y.

Problem 6.10-1 Determine the shape factor f for a cross section in the shape of a double trapezoid having the dimensions shown in the figure.

Also, check your result for the special cases of a rhombus ($b_1 = 0$) and a rectangle ($b_1 = b_2$).



Solution 6.10-1 **Double trapezoid**



Neutral axis passes through the centroid C.

Use case 8, Appendix D.

SECTION MODULUS S

$$I_z = 2\left(\frac{h}{2}\right)^3 (3b_1 + b_2)/12$$

$$= \frac{h^3}{48}(3b_1 + b_2)$$

$$C = h/2 \quad S = \frac{I}{C} = \frac{h^2}{24}(3b_1 + b_2)$$

PLASTIC MODULUS Z (EQ. 6-78)

$$A = 2\left(\frac{h}{2}\right)(b_1 + b_2)/2 = \frac{h}{2}(b_1 + b_2)$$

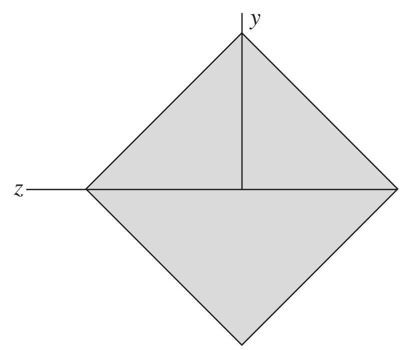
$$\bar{y}_1 = \bar{y}_2 = \frac{1}{3}\left(\frac{h}{2}\right)\left(\frac{2b_1 + b_2}{b_1 + b_2}\right)$$

$$z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{h^2}{12}(2b_1 + b_2)$$

SHAPE FACTOR f (EQ. 6.79)

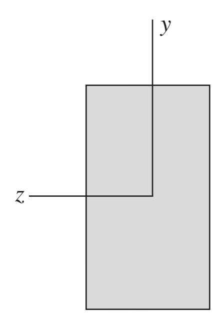
$$F = \frac{z}{S} = \frac{2(2b_1 + b_2)}{3b_1 + b_2} \quad \leftarrow$$

SPECIAL CASE – RHOMBUS



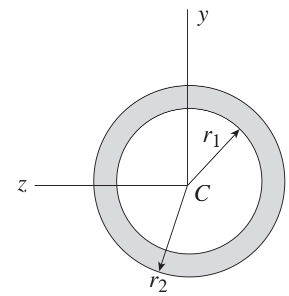
$b_1 = 0 \quad f = 2$

SPECIAL CASE – RECTANGLE

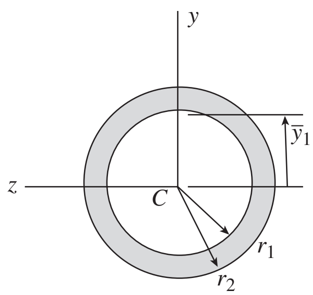


$b_1 = b_2 \quad f = \frac{3}{2}$

Problem 6.10-2 (a) Determine the shape factor f for a hollow circular cross section having inner radius r_1 and outer radius r_2 (see figure).
(b) If the section is very thin, what is the shape factor?



Solution 6.10-2 Hollow circular cross sections



Neutral axis passes through the centroid C.
Use cases 9 and 10, Appendix D.

SECTION MODULUS S

$$I_z = \frac{\pi}{4}(r_2^4 - r_1^4) \quad c = r_2$$

$$S = \frac{I_z}{c} = \frac{\pi}{4r_2}(r_2^4 - r_1^4)$$

PLASTIC MODULUS Z (EQ. 6-78)

$$A = \pi(r_2^2 - r_1^2) \quad \text{For a semicircle,} \quad \bar{y} = \frac{4r}{3\pi}$$

$$\bar{y}_1 = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{\left(\frac{4r_2}{3\pi}\right)\left(\frac{\pi r_2^2}{2}\right) - \left(\frac{4r_1}{3\pi}\right)\left(\frac{\pi r_1^2}{2}\right)}{\pi/2(r_2^2 - r_1^2)}$$

$$= \frac{4}{3\pi}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

$$\bar{y}_1 = \bar{y}_2 \quad z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{4}{3}(r_2^3 - r_1^3)$$

(a) SHAPE FACTOR f (EQ. 6-79)

$$f = \frac{z}{S} = \frac{16r_2(r_2^3 - r_1^3)}{3\pi(r_2^4 - r_1^4)} \quad \longleftarrow$$

(b) THIN SECTION ($r_1 \to r_2$)

Rewrite the expression for the shape factor f.

$$(r_2^3 - r_1^3) = (r_2 - r_1)(r_2^2 + r_1 r_2 + r_1^2)$$

$$(r_2^4 - r_1^4) = (r_2 - r_1)(r_2 + r_1)(r_2^2 + r_1^2)$$

$$f = \frac{16r_2}{3\pi}\left[\frac{r_2^2 + r_1 r_2 + r_1^2}{(r_2 + r_1)(r_2^2 + r_1^2)}\right]$$

$$= \frac{16}{3\pi}\left[\frac{1 + r_1/r_2 + (r_1/r_2)^2}{(1 + r_1/r_2)(1 + r_1^2/r_2^2)}\right]$$

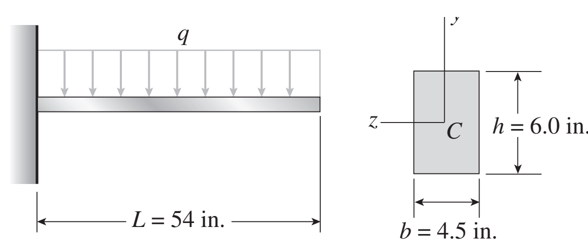
Let $r_1/r_2 \to 1$ $\quad f = \frac{16}{3\pi}\left(\frac{3}{4}\right) = \frac{4}{\pi} \approx 1.27 \quad \longleftarrow$

SPECIAL CASE OF A SOLID CIRCULAR CROSS SECTION

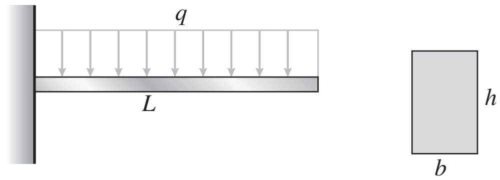
Let $r_1 = 0 \quad f = \frac{16}{3\pi}\left(\frac{1}{1}\right) = \frac{16}{3\pi}$ (Eq. 6-90)

Problem 6.10-3 A cantilever beam of length $L = 54$ in. supports a uniform load of intensity q (see figure). The beam is made of steel ($\sigma_Y = 36$ ksi) and has a rectangular cross section of width $b = 4.5$ in. and height $h = 6.0$ in.

What load intensity q will produce a fully plastic condition in the beam?



Solution 6.10-3 **Cantilever beam (rectangular cross section)**



MAXIMUM BENDING MOMENT: $M_{max} = \frac{qL^2}{2}$

PLASTIC MOMENT: $M_P = \frac{\sigma_y bh^2}{4}$

$M_{max} = M_P \quad \frac{qL^2}{2} = \frac{\sigma_y bh^2}{4} \quad q = \frac{\sigma_y bh^2}{2L^2}$ ←

SUBSTITUTE NUMERICAL DATA:

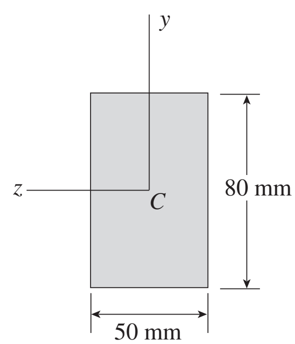
$L = 54$ in. $\sigma_Y = 36$ ksi
$b = 4.5$ in. $h = 6.0$ in.

$\therefore q = 1000$ lb/in. ←

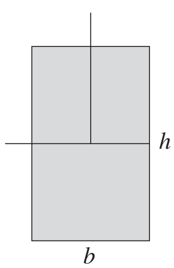
Problem 6.10-4 A steel beam of rectangular cross section is 50 mm wide and 80 mm high (see figure). The yield stress of the steel is 210 MPa.

(a) What percent of the cross-sectional area is occupied by the elastic core if the beam is subjected to a bending moment of 13.0 kN · m acting about the z axis?

(b) What is the magnitude of the bending moment that will cause 50% of the cross section to yield?



Solution 6.10-4 **Rectangular cross section**



$b = 50$ mm
$h = 80$ mm
$\sigma_Y = 210$ MPa

(a) ELASTIC CORE ($M = 13.0$ kN · m)

$M_Y = \frac{\sigma_Y bh^2}{6} = 11{,}200$ N · m

$M_P = \frac{\sigma_Y bh^2}{4} = 16{,}800$ N · m

M is between M_Y and M_P.

Eq. (6-85): $e = h\sqrt{\frac{1}{2}\left(\frac{3}{2} - \frac{M}{M_y}\right)} = 32.950$ mm

Percent of cross-sectional area is

$\frac{2e}{h}(100) = \frac{65.90\,(100)}{80}$

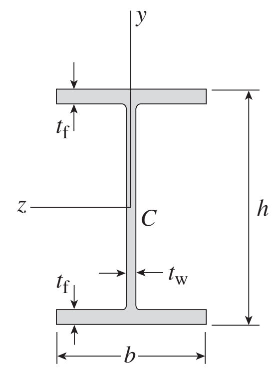
$= 82.4\ \%$ ←

(b) ELASTIC CORE $\left(e = \frac{h}{4} = 20\ \text{MM}\right)$

Eq. (6-84): $M = M_Y\left(\frac{3}{2} - \frac{2e^2}{h^2}\right)$

$= 15.4$ kN · m ←

Problem 6.10-5 Calculate the shape factor f for the wide-flange beam shown in the figure if $h = 12.0$ in., $b = 6.0$ in., $t_f = 0.6$ in., and $t_w = 0.4$ in.



Probs. 6.10-5 and 6.10-6

Solution 6.10-5 Wide-flange beam

$h = 12.0$ in. $b = 6.0$ in. $t_f = 0.6$ in. $t_w = 0.4$ in.

SECTION MODULUS ($S = I/c$)

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b - t_w)(h - 2t_f)^3$$
$$= 276.1 \text{ in.}^4$$
$$c = \frac{h}{2} = 6.0 \text{ in.} \qquad S = \frac{I}{c} = 96.0 \text{ in.}^3$$

PLASTIC MODULUS (EQ. 6-86)

$$z = \frac{1}{4}[bh^2 - (b - t_w)(h - 2t_f)^2] = 52.7 \text{ in.}^3$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{z}{S} = 1.15 \quad \longleftarrow$$

Problem 6.10-6 Solve the preceding problem for a wide-flange beam with $h = 400$ mm, $b = 150$ mm, $t_f = 12$ mm, and $t_w = 8$ mm.

Solution 6.10-6 Wide-flange beam

$h = 400$ mm $b = 150$ mm $t_f = 12$ mm $t_w = 8$ mm

SECTION MODULUS ($S = I/c$)

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b - t_w)(h - 2t_f)^3$$
$$= 171.0 \times 10^6 \text{ mm}^4$$
$$c = \frac{h}{2} = 200 \text{ mm} \qquad S = \frac{I}{c} = 854.9 \times 10^3 \text{ mm}^3$$

PLASTIC MODULUS (EQ. 6-86)

$$z = \frac{1}{4}[bh^2 - (b - t_w)(h - 2t_f)^2]$$
$$= 981.2 \times 10^3 \text{ mm}^3$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{z}{S} = 1.15 \quad \longleftarrow$$

Problem 6.10-7 Determine the plastic modulus Z and shape factor f for a W 10 × 30 wide-flange beam. (*Note:* Obtain the cross-sectional dimensions and section modulus of the beam from Table E-1, Appendix E.)

Solution 6.10-7 Wide-flange beam

W10 × 30
$h = 10.47$ in. $b = 5.810$ in.
$t_f = 0.510$ in. $t_w = 0.300$ in. $S = 32.4 \text{ in.}^3$

PLASTIC MODULUS (EQ. 6-86)

$$Z = \frac{1}{4}[bh^2 - (b - t_w)(h - 2t_f)^2] = 36.21 \text{ in.}^3 \quad \longleftarrow$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{Z}{S} = 1.12 \quad \longleftarrow$$

Problem 6.10-8 Solve the preceding problem for a W 8 × 28 wide-flange beam.

Solution 6.10-8 Wide-flange beam

W 8 × 28

$h = 8.06$ in. $b = 6.535$ in.

$t_f = 0.465$ in. $t_w = 0.285$ in. $S = 24.3$ in.^{3}

PLASTIC MODULUS (EQ. 6-86)

$$Z = \frac{1}{4}[bh^2 - (b - t_w)(h - 2t_f)^2] = 26.70 \text{ in.}^3 \longleftarrow$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{Z}{S} = 1.10 \longleftarrow$$

Problem 6.10-9 Determine the yield moment M_Y, plastic moment M_P, and shape factor f for a W 16 × 77 wide-flange beam if $\sigma_Y = 36$ ksi. (*Note:* Obtain the cross-sectional dimensions and section modulus of the beam from Table E-1, Appendix E.)

Solution 6.10-9 Wide-flange beam

W 16 × 77 $h = 16.52$ in. $b = 10.295$ in.

$t_f = 0.760$ in. $t_w = 0.455$ in. $\sigma_Y = 36$ ksi

$S = 134$ in.^{3}

YIELD MOMENT (EQ. 6-74)

$$M_Y = \sigma_Y S = 4820 \text{ k-in.} \longleftarrow$$

PLASTIC MODULUS (EQ. 6-86)

$$Z = \frac{1}{4}[bh^2 - (b - t_w)(h - 2t_f)^2] = 148.9 \text{ in.}^3$$

PLASTIC MOMENT (EQ. 6-77)

$$M_P = \sigma_Y Z = 5360 \text{ k-in.} \longleftarrow$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{M_P}{M_Y} = 1.11 \longleftarrow$$

Problem 6.10-10 Solve the preceding problem for a W 10 × 45 wide-flange beam.

Solution 6.10-10 Wide-flange beam

W 10 × 45 $h = 10.10$ in. $b = 8.020$ in.

$t_f = 0.620$ in. $t_w = 0.350$ in. $\sigma_Y = 36$ ksi

$S = 49.1$ in.^{3}

YIELD MOMENT (EQ. 6-74)

$$M_Y = \sigma_Y S = 1770 \text{ k-in.} \longleftarrow$$

PLASTIC MODULUS (EQ. 6-86)

$$Z = \frac{1}{4}[bh^2 - (b - t_w)(h - 2t_f)^2]$$
$$= 54.0 \text{ in.}^3$$

PLASTIC MOMENT (EQ. 6-77)

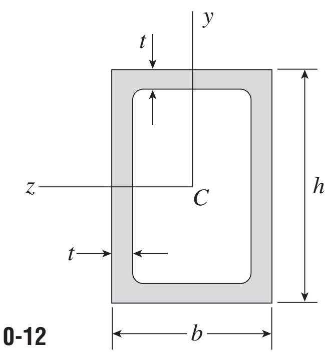
$$M_P = \sigma_Y z = 1940 \text{ k-in.} \longleftarrow$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{M_P}{M_Y} = 1.10 \longleftarrow$$

Problem 6.10-11 A hollow box beam with height $h = 16$ in., width $b = 8$ in., and constant wall thickness $t = 0.75$ in. is shown in the figure. The beam is constructed of steel with yield stress $\sigma_Y = 32$ ksi.

Determine the yield moment M_Y, plastic moment M_P, and shape factor f.



Probs. 6.10-11 and 6.10-12

Solution 6.10-11 **Hollow box beam**

$h = 16$ in. $b = 8$ in.
$t = 0.75$ in. $\sigma_Y = 32$ ksi

SECTION MODULUS ($S = I/c$)

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b - 2t)(h - 2t)^3$$
$$= 1079 \text{ in.}^4$$

$$c = \frac{h}{2} = 8.0 \text{ in.} \qquad S = \frac{I}{c} = 134.9 \text{ in.}^3$$

YIELD MOMENT (EQ. 6-74)

$$M_Y = \sigma_Y S = 4320 \text{ k-in.} \quad \longleftarrow$$

PLASTIC MODULUS

use (Eq. 6-86) with $t_w = 2t$ and $t_f = t$:

$$Z = \frac{1}{4}[bh^2 - (b - 2t)(h - 2t)^2]$$
$$= 170.3 \text{ in.}^3$$

PLASTIC MOMENT (EQ. 6-77)

$$M_p = \sigma_y z = 5450 \text{ k-in.} \quad \longleftarrow$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{M_P}{M_Y} = \frac{Z}{S} = 1.26 \quad \longleftarrow$$

Problem 6.10-12 Solve the preceding problem for a box beam with dimensions $h = 0.4$ m, $b = 0.2$ m, and $t = 20$ mm. The yield stress of the steel is 230 MPa.

Solution 6.10-12 **Hollow box beam**

$h = 400$ mm $b = 200$ mm
$t = 20$ mm $\sigma_Y = 230$ MPa

SECTION MODULUS ($S = I/c$)

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b - 2t)(h - 2t)^3$$
$$= 444.6 \times 10^6 \text{ mm}^4$$

$$c = \frac{h}{2} = 200 \text{ mm} \qquad S = \frac{I}{c} = 2.223 \times 10^6 \text{ mm}^3$$

YIELD MOMENT (EQ. 6-74)

$$M_Y = \sigma_y S = 511 \text{ kN} \cdot \text{m} \quad \longleftarrow$$

PLASTIC MODULUS

use (Eq. 6-86) with $t_w = 2t$ and $t_f = t$:

$$Z = \frac{1}{4}[bh^2 - (b - 2t)(h - 2t)^2]$$
$$= 2.816 \times 10^6 \text{ mm}^3$$

PLASTIC MOMENT (EQ. 6-77)

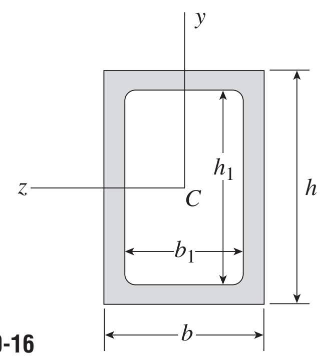
$$M_P = \sigma_Y z = 648 \text{ kN} \cdot \text{m} \quad \longleftarrow$$

SHAPE FACTOR (Eq. 6-79)

$$f = \frac{Z}{S} = 1.27 \quad \longleftarrow$$

Problem 6.10-13 A hollow box beam with height $h = 9.0$ in., inside height $h_1 = 7.5$ in., width $b = 5.0$ in., and inside width $b_1 = 4.0$ in. is shown in the figure.

Assuming that the beam is constructed of steel with yield stress $\sigma_Y = 33$ ksi, calculate the yield moment M_Y, plastic moment M_P, and shape factor f.



Probs. 6.10-13 through 6.10-16

Solution 6.10-13 **Hollow box beam**

$h = 9.0$ in. $b = 5.0$ in.
$h_1 = 7.5$ in. $b_1 = 4.0$ in. $\sigma_Y = 33$ ksi

SECTION MODULUS ($S = I/c$)

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 163.12 \text{ in.}^4$$

$$c = \frac{h}{2} = 4.5 \text{ in.} \qquad S = \frac{I}{c} = 36.25 \text{ in.}^3$$

YIELD MOMENT (EQ. 6-74)

$$M_Y = \sigma_Y S = 1196 \text{ k-in.} \longleftarrow$$

PLASTIC MODULUS

use (Eq. 6-86) with $b - t_w = b_1$ and $h - 2t_f = h_1$:

$$Z = \frac{1}{4}(bh^2 - b_1h_1^2) = 45.0 \text{ in.}^3$$

PLASTIC MOMENT (EQ. 6-77)

$$M_P = \sigma_Y z = 1485 \text{ k-in.} \longleftarrow$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{M_P}{M_Y} = \frac{Z}{S} = 1.24 \longleftarrow$$

Problem 6.10-14 Solve the preceding problem for a box beam with dimensions $h = 200$ mm, $h_1 = 160$ mm, $b = 150$ mm, and $b_1 = 130$ mm. Assume that the beam is constructed of steel with yield stress $\sigma_Y = 220$ MPa.

Solution 6.10-14 **Hollow box beam**

$h = 200$ mm $b = 150$ mm
$h_1 = 160$ mm $b_1 = 130$ mm $\sigma_Y = 220$ MPa

SECTION MODULUS ($S = I/c$)

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 55.63 \times 10^6 \text{ mm}^4$$

$$c = \frac{h}{2} = 100 \text{ mm} \qquad S = \frac{I}{c} = 556.3 \times 10^3 \text{ mm}^3$$

YIELD MOMENT (EQ. 6-74)

$$M_Y = \sigma_Y S = 122 \text{ kN} \cdot \text{m} \longleftarrow$$

PLASTIC MODULUS

use (Eq. 6-86) with $b - t_w = b_1$ and $h - 2t_f = h_1$:

$$Z = \frac{1}{4}(bh^2 - b_1h_1^2) = 668.0 \times 10^3 \text{ mm}^3$$

PLASTIC MOMENT (EQ. 6-77)

$$M_P = \sigma_Y Z = 147 \text{ kN} \cdot \text{m} \longleftarrow$$

SHAPE FACTOR (EQ. 6-79)

$$f = \frac{M_P}{M_Y} = \frac{Z}{S} = 1.20 \longleftarrow$$

Problem 6.10-15 The hollow box beam shown in the figure is subjected to a bending moment M of such magnitude that the flanges yield but the webs remain linearly elastic.

(a) Calculate the magnitude of the moment M if the dimensions of the cross section are $h = 14$ in., $h_1 = 12.5$ in., $b = 8$ in., and $b_1 = 7$ in. Also, the yield stress is $\sigma_Y = 42$ ksi.

(b) What percent of the moment M is produced by the elastic core?

Solution 6.10-15 **Hollow box beam**

$h = 14$ in. $\quad b = 8$ in.

$h_1 = 12.5$ in. $\quad b_1 = 7$ in. $\quad \sigma_Y = 42$ ksi

(see Figure 6-47, Example 6-9)

ELASTIC CORE

$$S_1 = \frac{1}{6}(b - b_1)h_1^2 = 26.04 \text{ in.}^3$$

$$M_1 = \sigma_Y S_1 = 1094 \text{ k-in.}$$

PLASTIC FLANGES

F = force in one flange

$$F = \sigma_Y b\left(\frac{1}{2}\right)(h - h_1) = 252.0 \text{ k}$$

$$M_2 = F\left(\frac{h + h_1}{2}\right) = 3339 \text{ k-in.}$$

(a) BENDING MOMENT

$$M = M_1 + M_2 = 4430 \text{ k-in.} \quad \longleftarrow$$

(b) PERCENT DUE TO ELASTIC CORE

$$\text{Percent} = \frac{M_1}{M}(100) = 25\% \quad \longleftarrow$$

Problem 6.10-16 Solve the preceding problem for a box beam with dimensions $h = 400$ mm, $h_1 = 360$ mm, $b = 200$ mm, and $b_1 = 160$ mm, and with yield stress $\sigma_Y = 220$ MPa.

Solution 6.10-16 **Hollow box beam**

$h = 400$ mm $\quad b = 200$ mm

$h_1 = 360$ mm $\quad b_1 = 160$ mm $\quad \sigma_Y = 220$ MPa

(see Figure 6-47, Example 6-9)

ELASTIC CORE

$$S_1 = \frac{1}{6}(b - b_1)h_1^2 = 864 \times 10^3 \text{ mm}^3$$

$$M_1 = \sigma_Y S_1 = 190.1 \text{ kN} \cdot \text{m}$$

PLASTIC FLANGES

F = force in one flange

$$F = \sigma_Y b\left(\frac{1}{2}\right)(h - h_1) = 880.0 \text{ kN}$$

$$M_2 = F\left(\frac{h + h_1}{2}\right) = 334.4 \text{ kN} \cdot \text{m}$$

(a) BENDING MOMENT

$$M = M_1 + M_2 = 524 \text{ kN} \cdot \text{m} \quad \longleftarrow$$

(b) PERCENT DUE TO ELASTIC CORE

$$\text{Percent} = \frac{M_1}{M}(100) = 36\% \quad \longleftarrow$$

Problem 6.10-17 A W 12 × 50 wide-flange beam is subjected to a bending moment M of such magnitude that the flanges yield but the web remains linearly elastic.

(a) Calculate the magnitude of the moment M if the yield stress is $\sigma_Y = 36$ ksi.

(b) What percent of the moment M is produced by the elastic core?

Solution 6.10-17 Wide-flange beam

W 12 × 50 $h = 12.19$ in. $b = 8.080$ in.
$t_f = 0.640$ in. $t_w = 0.370$ in. $\sigma_Y = 36$ ksi

ELASTIC CORE

$$S_1 = \frac{1}{6} t_w (h - 2t_f)^2 = 7.340 \text{ in.}^3$$

$$M_1 = \sigma_Y S_1 = 264.2 \text{ k-in.}$$

PLASTIC FLANGES

F = force in one flange

$$F = \sigma_Y b t_f = 186.2 \text{ k}$$

$$M_2 = F(h - t_f) = 2151 \text{ k-in.}$$

(a) BENDING MOMENT

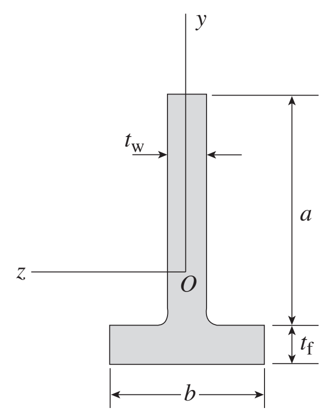
$$M = M_1 + M_2 = 2410 \text{ k-in.} \quad \longleftarrow$$

(b) PERCENT DUE TO ELASTIC CORE

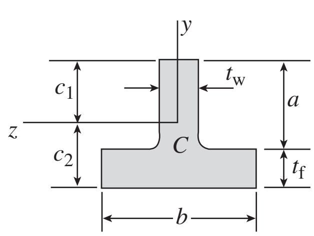
$$\text{Percent} = \frac{M_1}{M}(100) = 11\% \quad \longleftarrow$$

Problem 6.10-18 A singly symmetric beam of T-section (see figure) has cross-sectional dimensions $b = 140$ mm, $a = 200$ mm, $t_w = 20$ mm, and $t_f = 25$ mm.

Calculate the plastic modulus Z and the shape factor f.



Solution 6.10-18 Beam of T-section



$b = 140$ mm $a = 200$ mm
$t_w = 20$ mm $t_f = 25$ mm

ELASTIC BENDING

$$c_2 = \frac{\sum y_i A_i}{\sum A_i} = \frac{\left(\frac{t_f}{2}\right)(bt_f) + \left(\frac{a}{2} + t_f\right)(at_w)}{bt_f + at_w}$$

$$= 78.50 \text{ mm}$$

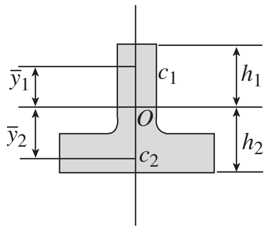
$$c_1 = a + t_f - c_2 = 152.50 \text{ mm}$$

$$I_z = \frac{1}{3} t_w c_1^3 + \frac{1}{3} b c_2^3 - \frac{1}{3}(b - t_w)(c_2 - t_f)^3$$

$$= 37.14 \times 10^6 \text{ mm}^4$$

$$S = \frac{I}{c_1} = 243.5 \times 10^3 \text{ mm}^3$$

PLASTIC BENDING



$$A = bt_f + at_w = 7500 \text{ mm}^2$$

$$h_1 t_w = \frac{A}{2}$$

$$h_1 = 187.5 \text{ mm}$$

$$h_2 = a + t_f - h_1 = 37.5 \text{ mm}$$

$$\bar{y}_1 = \frac{h_1}{2} = 93.75 \text{ mm}$$

$$\bar{y}_2 = \frac{\sum y_i A_i}{A/2}$$

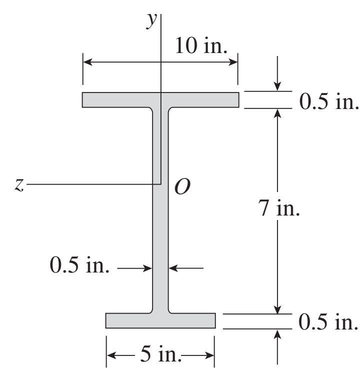
$$= \frac{\frac{1}{2}bh_2^2 - \frac{1}{2}(b - t_w)(h_2 - t_f)^2}{A/2} = 23.75 \text{ mm}$$

$$Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = 441 \times 10^3 \text{ mm}^3 \quad \longleftarrow$$

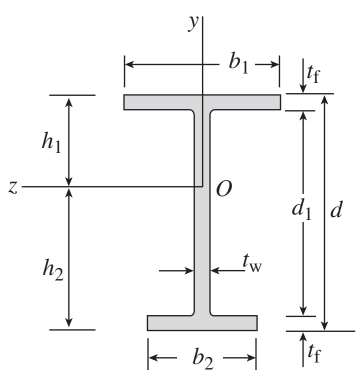
$$f = \frac{Z}{S} = 1.81 \quad \longleftarrow$$

Problem 6.10-19 A wide-flange beam of unbalanced cross section has the dimensions shown in the figure.

Determine the plastic moment M_P if $\sigma_Y = 36$ ksi.



Solution 6.10-19 **Unbalanced wide-flange beam**



$\sigma_Y = 36$ ksi $\quad b_1 = 10$ in. $\quad b_2 = 5$ in.
$t_w = 0.5$ in. $\quad d = 8$ in. $\quad d_1 = 7$ in.
$t_f = 0.5$ in. $\quad A = b_1 t_f + b_2 t_f + d_1 t_w = 11.0 \text{ in.}^2$

NEUTRAL AXIS UNDER FULLY PLASTIC CONDITIONS

$$\frac{A}{2} = h_1 t_w + (b_1 - t_w)t_f$$

from which we get $h_1 = 1.50$ in.
$h_2 = d - h_1 = 8.50$ in.

PLASTIC MODULUS

$$\bar{y}_1 = \frac{\sum y_i A_i}{A/2}$$

$$= \frac{(h_1/2)(t_w)(h_1) + (h_1 - t_f/2)(b_1 - t_w)(t_f)}{A/2}$$

$$= 1.182 \text{ in.}$$

$$\bar{y}_2 = \frac{\sum y_i A_i}{A/2}$$

$$= \frac{(h_2/2)(t_w)(h_2) + (h_2 - t_f/2)(b_2 - t_w)(t_f)}{A/2}$$

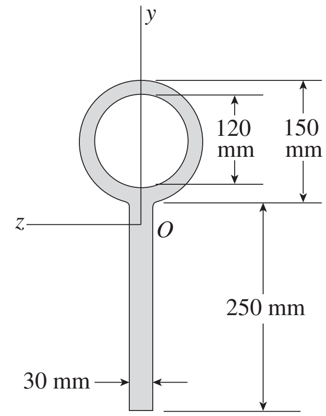
$$= 4.477 \text{ in.}$$

$$Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = 31.12 \text{ in.}^3$$

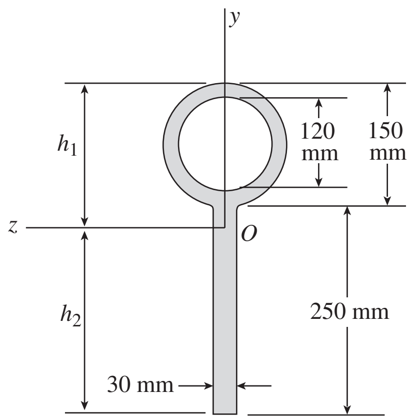
PLASTIC MOMENT

$$M_P = \sigma_y z = 1120 \text{ k-in.} \quad \longleftarrow$$

Problem 6.10-20 Determine the plastic moment M_P for a beam having the cross section shown in the figure if $\sigma_Y = 210$ MPa.



Solution 6.10-20 **Cross section of beam**



$\sigma_Y = 210$ MPa $\quad d_2 = 150$ mm $\quad d_1 = 120$ mm

NEUTRAL AXIS FOR FULLY PLASTIC CONDITIONS

Cross section is divided into two equal areas.

$$A = \frac{\pi}{4}[(150\text{ mm})^2 - (120\text{ mm})^2] + (250\text{ mm})(30\text{ mm}) = 13{,}862\text{ mm}^2$$

$$\frac{A}{2} = 6931\text{ mm}^2$$

$$(h_2)(30\text{ mm}) = \frac{A}{2} = 6931\text{ mm}^2$$

$$h_2 = 231.0\text{ mm}$$
$$h_1 = 150\text{ mm} + 250\text{ mm} - h_2 = 169.0\text{ mm}$$

PLASTIC MODULUS

$$\bar{y}_1 = \frac{\sum y_i A_i}{A/2} \text{ for upper half of cross section}$$

$$\bar{y}_2 = \frac{\sum y_i A_i}{A/2} \text{ for lower half of cross section}$$

$$Z = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = (\sum y_i A_i)_{\text{upper}} + (\sum y_i A_i)_{\text{lower}}$$

(Dimensions are in millimeters)

$$Z = (h_1 - 75)\left(\frac{\pi}{4}\right)(d_2^2 - d_1^2) + \left[\left(\frac{h_1 - 150}{2}\right)(30)(h_1 - 150)\right] + \left(\frac{h_2}{2}\right)(30)(h_2)$$

$$= 598{,}000 + 5{,}400 + 600{,}400$$
$$= 1404 \times 10^3\text{ mm}^3$$

PLASTIC MOMENT

$$M_P = \sigma_P Z = (210\text{ MPa})(1404 \times 10^3\text{ mm}^3)$$
$$= 295\text{ kN} \cdot \text{m} \quad \longleftarrow$$